

# Analyzing Risk through the Application of Six Sigma Concepts

## **Acknowledgment:**

This course applies *Six Sigma* concepts developed by Motorola, Inc. These concepts include associating an equivalent *sigma* with a given defect level, a measurement of customer perceived quality using defects per unit (DPU), and the Six Steps to Six Sigma.

## Objectives

- Familiarize participants with Six Sigma terminology and concepts needed for the course, “Six Sigma and Cost/Risk Management for S&T”
- Concepts include:
  - Process variation
  - Requirements and Capabilities
  - Variables & Attributes
  - Scorecard for Risk Analysis

# What is Six Sigma?

Six Sigma is a:

- method
- metric
- benchmark
- stretch goal

Six Sigma has become a metaphor for world-class quality.

## Definitions for Six Sigma

- Six Sigma is a way to measure the probability that a product being developed will have almost no risk.
- The probability of success is  $> 99.99966\%$  for each product characteristic.

## Definitions for Six Sigma

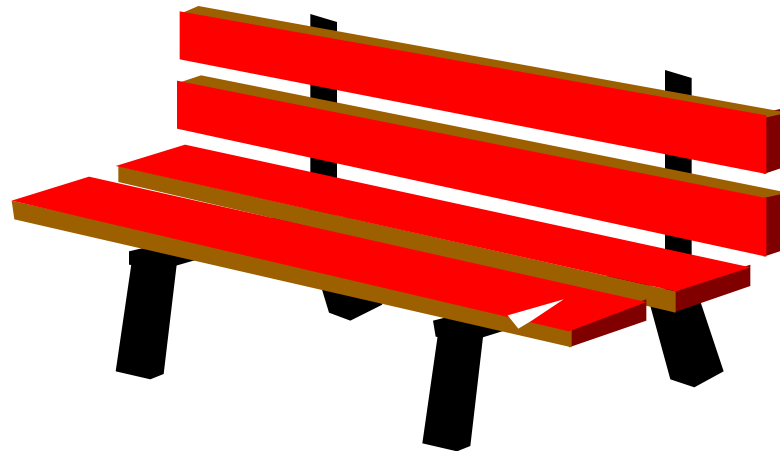
- Six Sigma is a way to measure the chance that a unit of product or a work process can be manufactured or performed with virtually zero defects.
- For variables, Six Sigma is  $C_p \geq 2$  AND  $C_{pk} \geq 1.5$
- For attributes, Six Sigma is no more than 3.4 defects per million.

## Why Six Sigma?

- The application of Six Sigma concepts to advanced technology projects increases the predictability of the result.
- The Six Sigma metrics allow us to assess the risk associated with prototype development and with production based on that prototype.
- Risk reduction helps us more effectively invest increasingly scarce development resources in a rapidly changing and competitive environment.

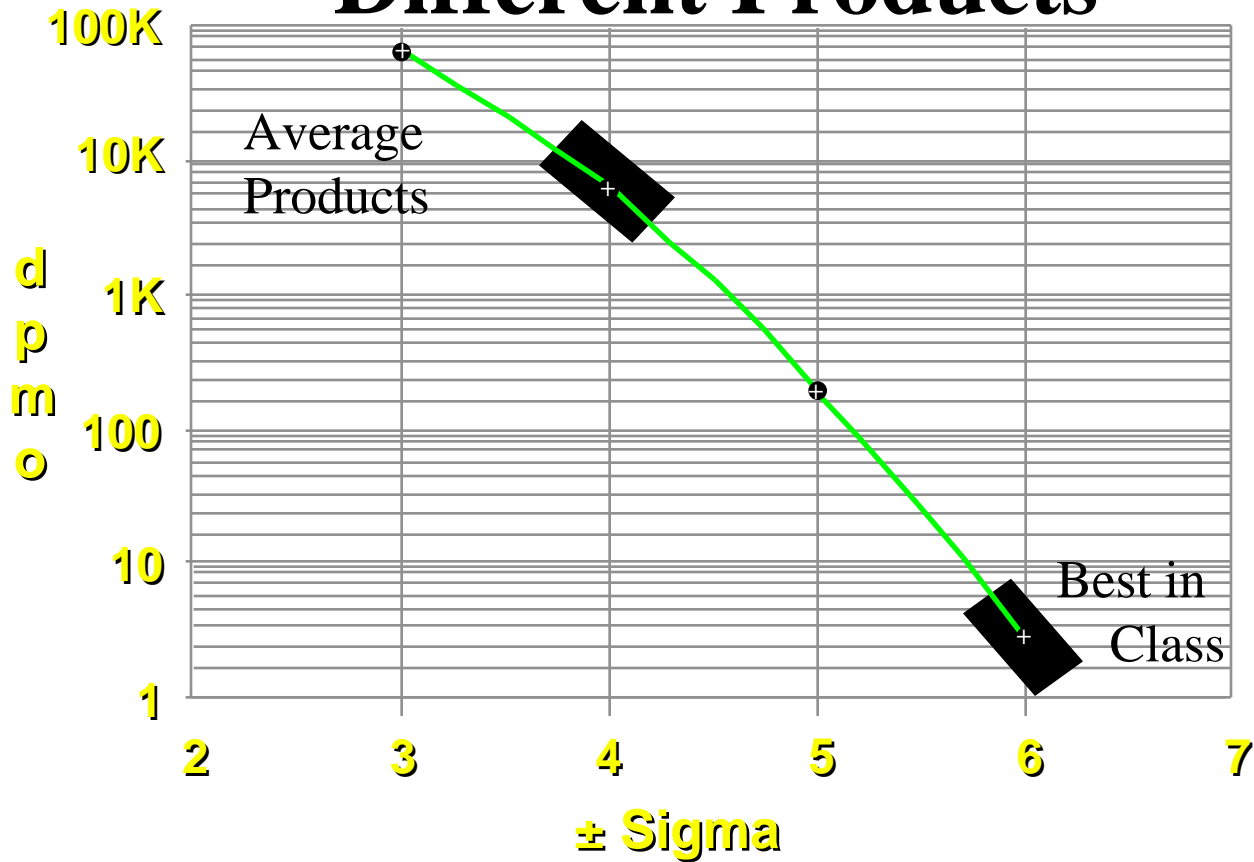
# Benchmarking

- Looking for the best, inside and outside of the organization
- Comparing our processes to the best
- Incorporating best practices into our processes





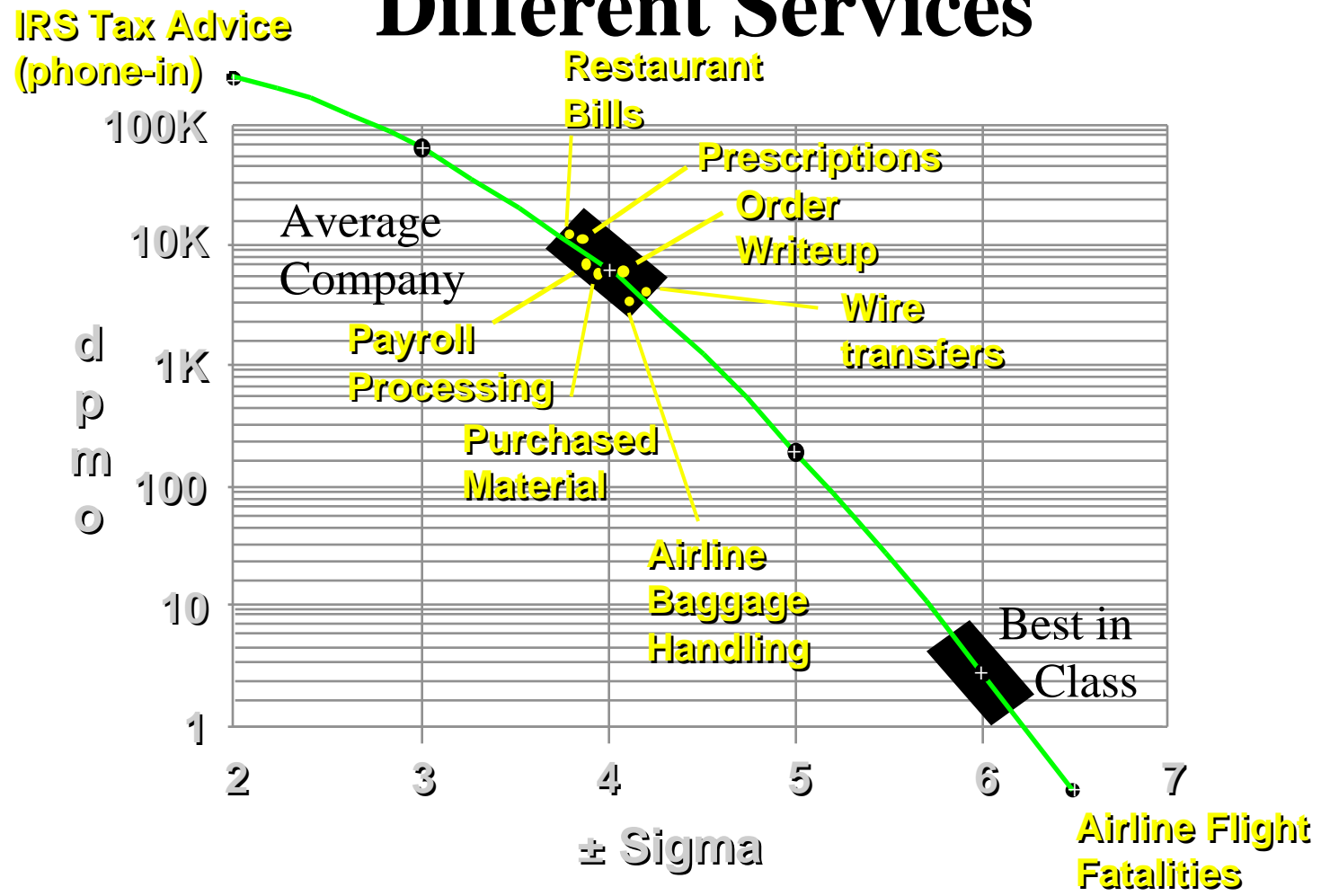
# Benchmarking Different Products



## Why Six Sigma?

- A Six Sigma manufacturing program will spend 1% or less of each dollar on the cost of non-conformance.
- A Four Sigma manufacturing program will spend as much as 25%
- A Four Sigma program cannot compete with a Six Sigma program.
- We believe program survival depends on achieving Six Sigma.

# Benchmarking Different Services



Acknowledgement: Motorola

Georgia Tech

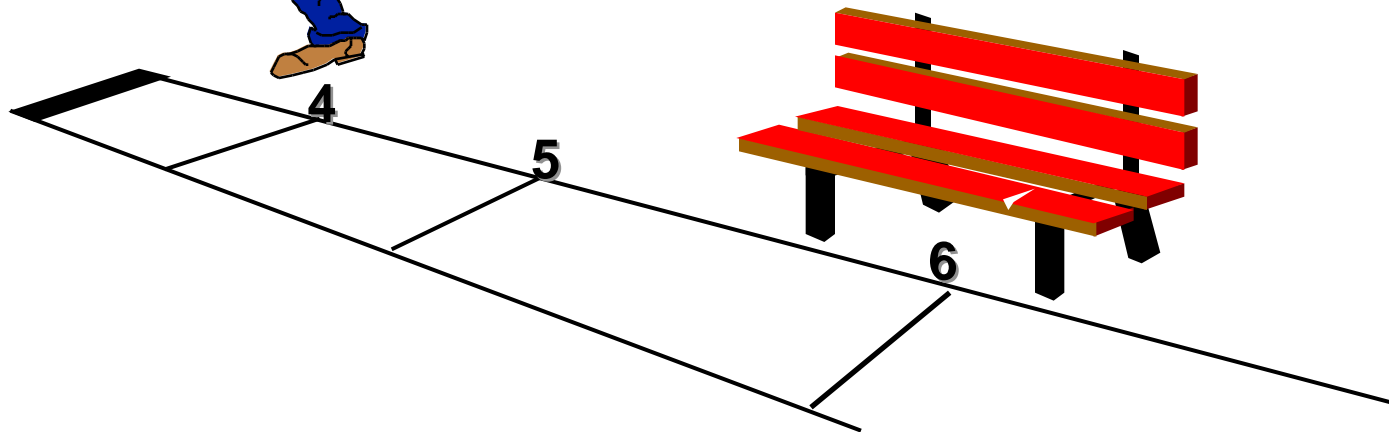
Texas Instruments

# A Stretch Goal

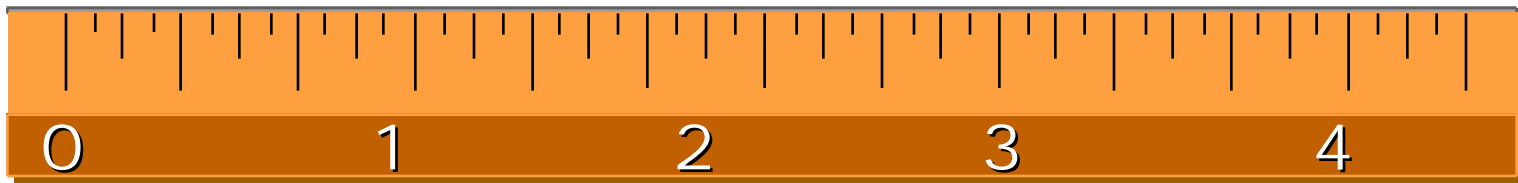
- **Six Sigma, the metric, quantifies the benchmark: “world class” quality**
- **Six Sigma, the method, is a set of principles to get to “world class”**



He won't get to the benchmark with his current process!



# Six Sigma - A Common Yardstick for Quality



# Six Sigma and the Product Life Cycle

Concept (6.1, 6.2)

Tech Transfer (6.3)

Production

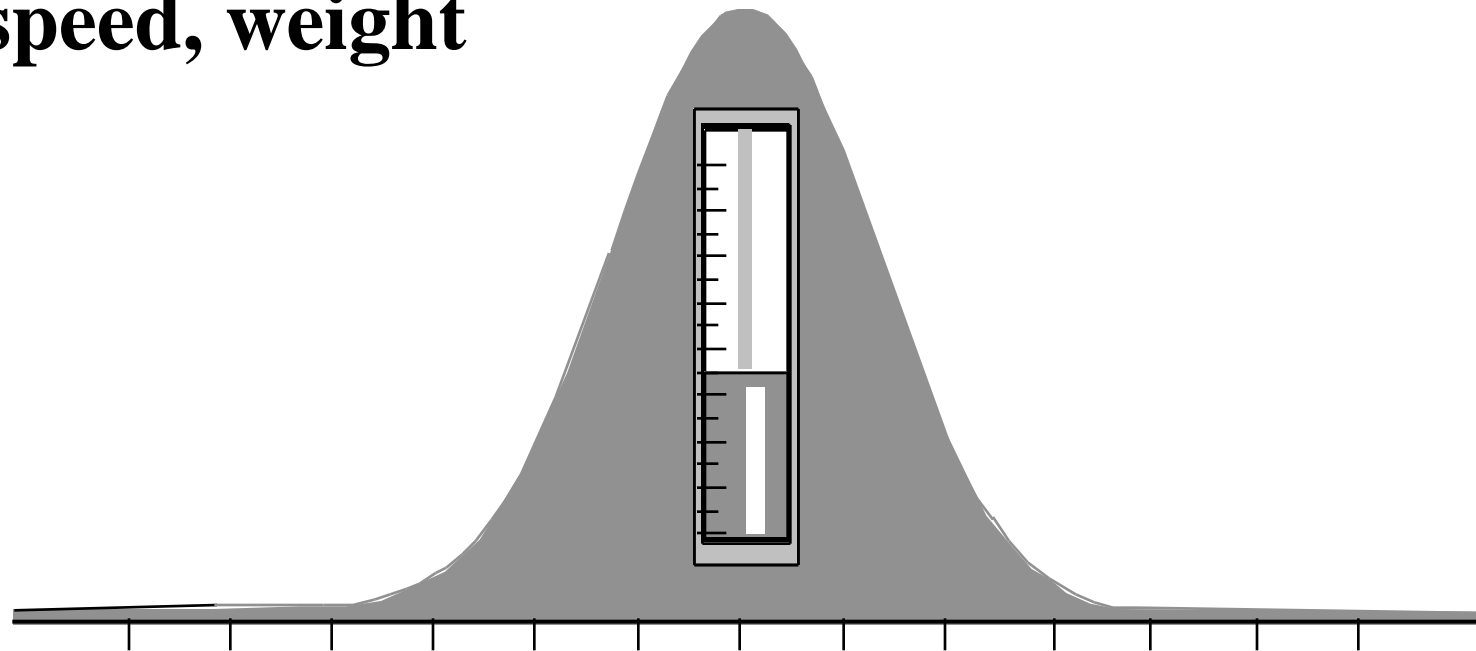
Six Sigma and  
Cost Risk Management  
in S&T

Six Sigma for  
Intellectual Activities

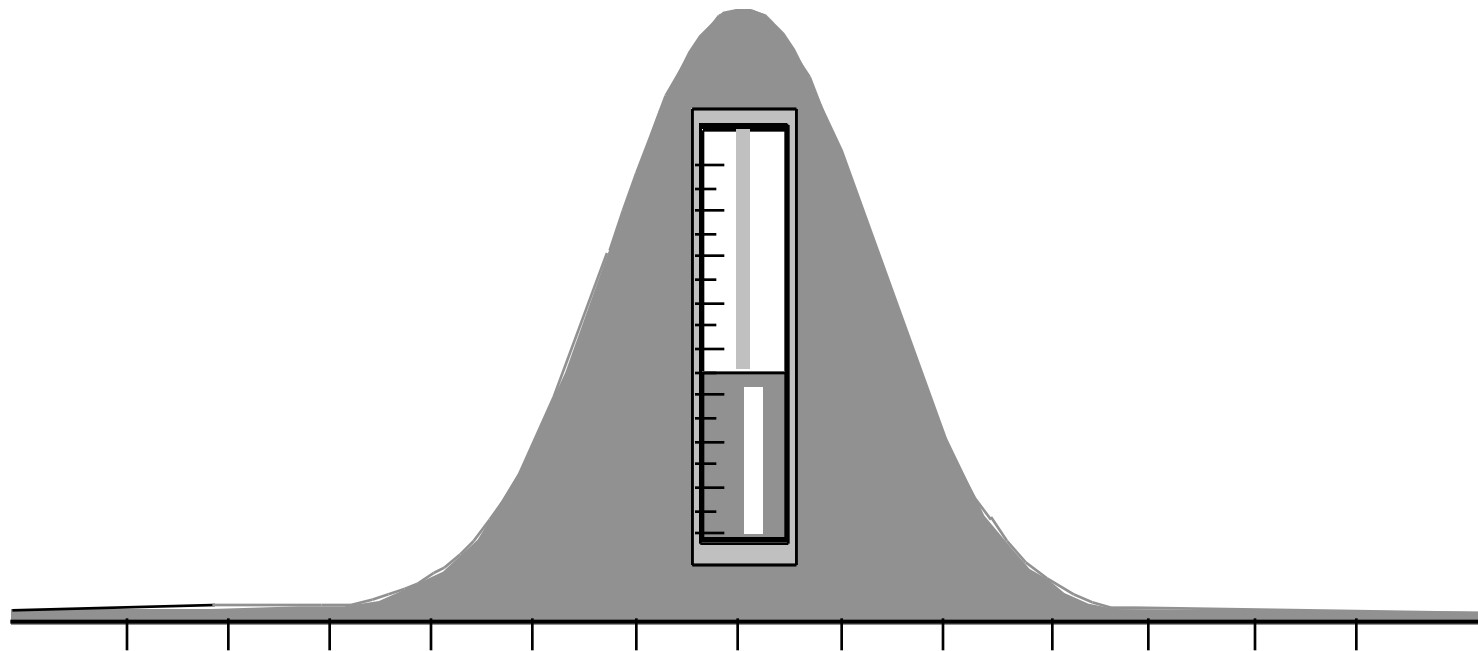
Design for  
Six Sigma Manufacturability

# Variables

are things measured on a continuous scale. Examples: temperature, volts, feet, speed, weight

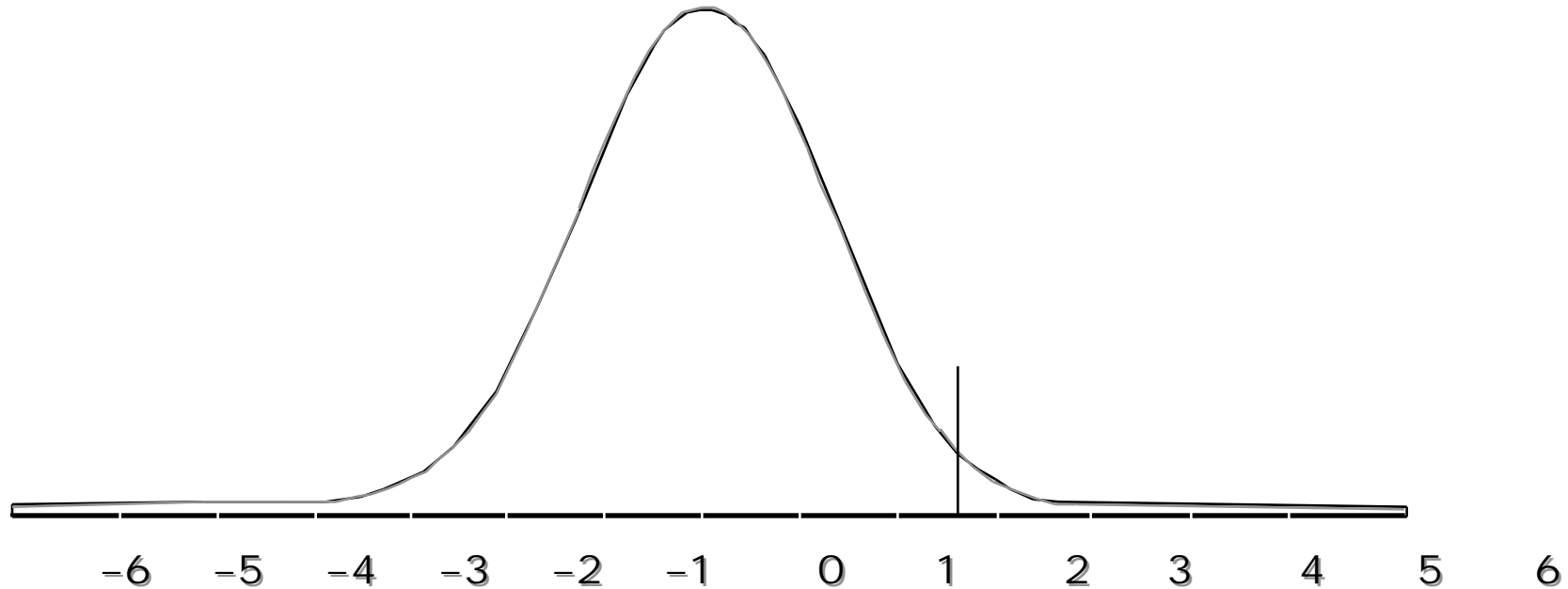


**Many product/process variables are distributed in ways that resemble the Normal distribution.**



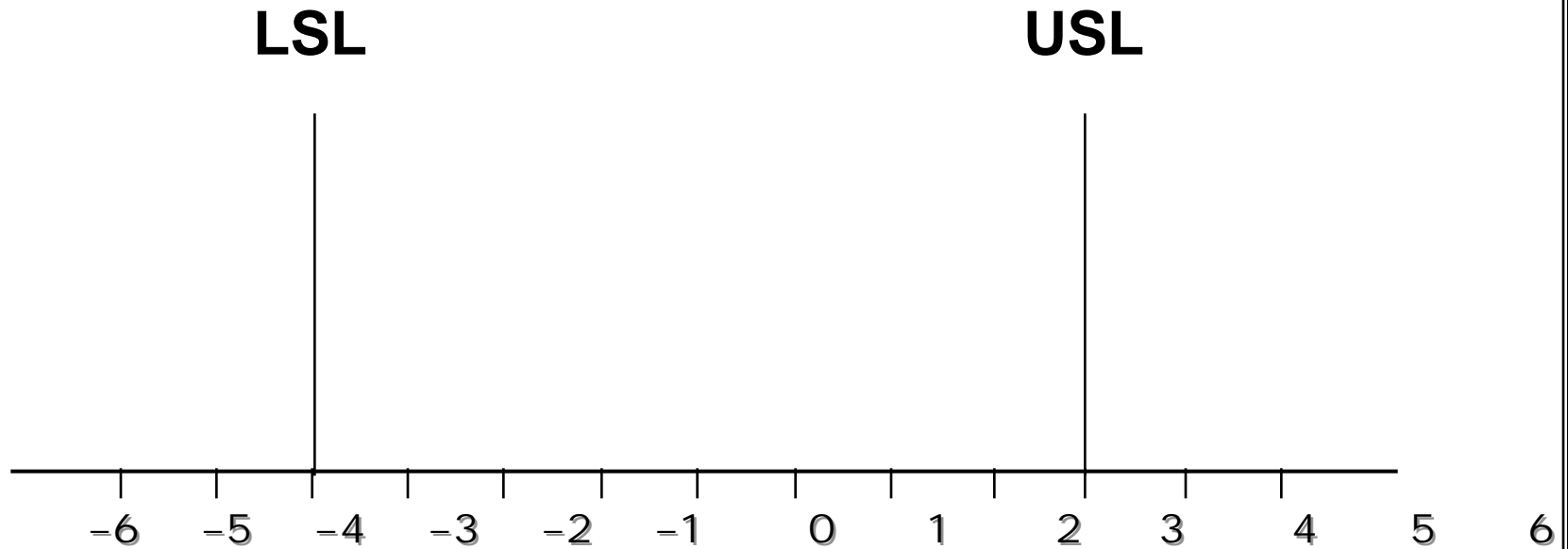


**The normal distribution  
is used to estimate the probability of  
exceeding a value.**



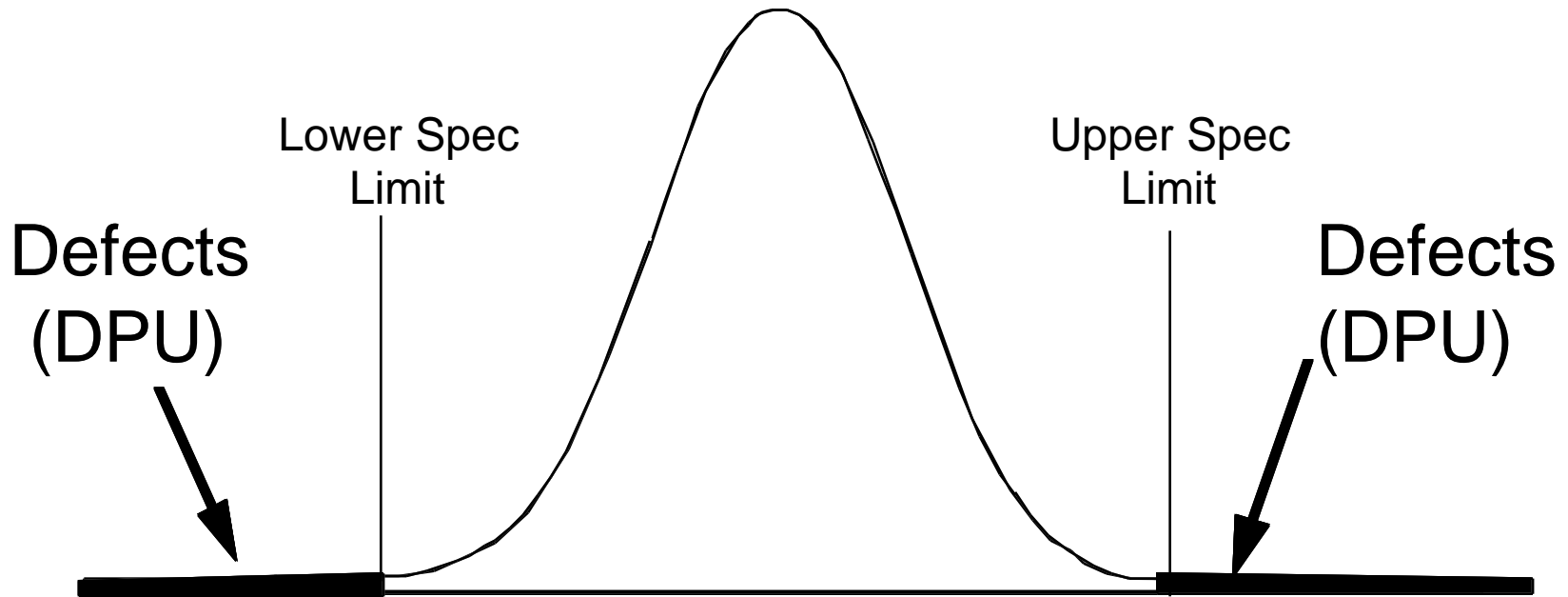
Probabilities for the normal distribution are found in Z tables.

# Specification Limits

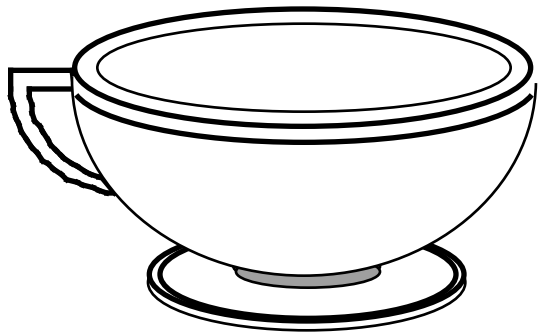


# What Causes Defects ?

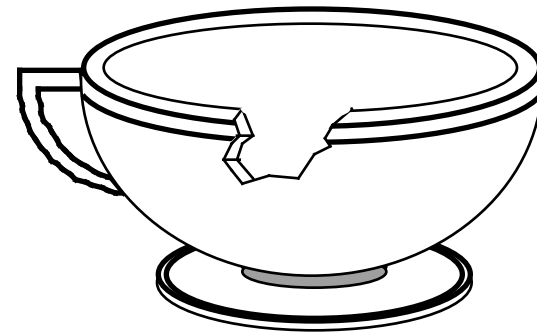
## The Capability does not meet the Requirement !



# Attributes



No Defect

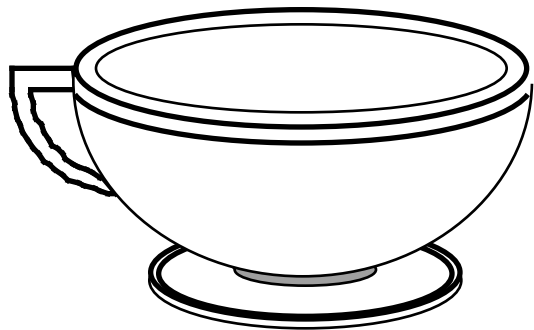


Defect

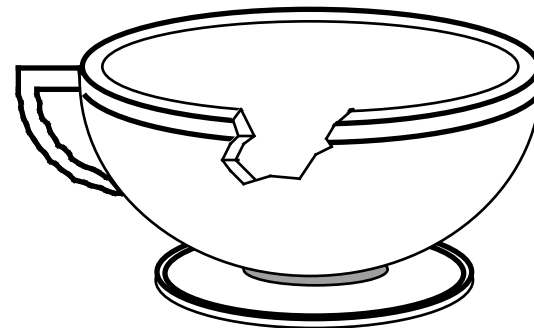
# Attributes

**What causes defects?**

**The capabilities do not meet the requirements.**

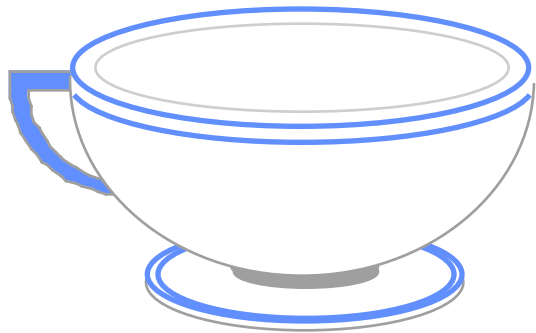


No Defect

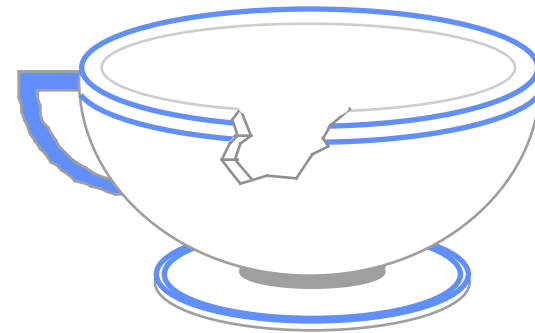


Defect

# Attributes Data



No Defect



Defect

$$\text{DPU} = \frac{\# \text{ defects found}}{\# \text{ units inspected}} = \frac{1}{200} = .005$$

## The probabilities are additive

- The defects (risk) that are estimated from product variables, and...
- the defects (risk) that are estimated from product attributes...
- can be added together...
- to form a metric.
- Defects per Unit (DPU)

**Customer**

# Metrics Cascade



**Defects (Risk)**



**Unit**



**Defects per Unit (DPU)**



**Opportunities/Unit**



**dpmo**



**Sigma Level**

For Benchmarking



# Defintion

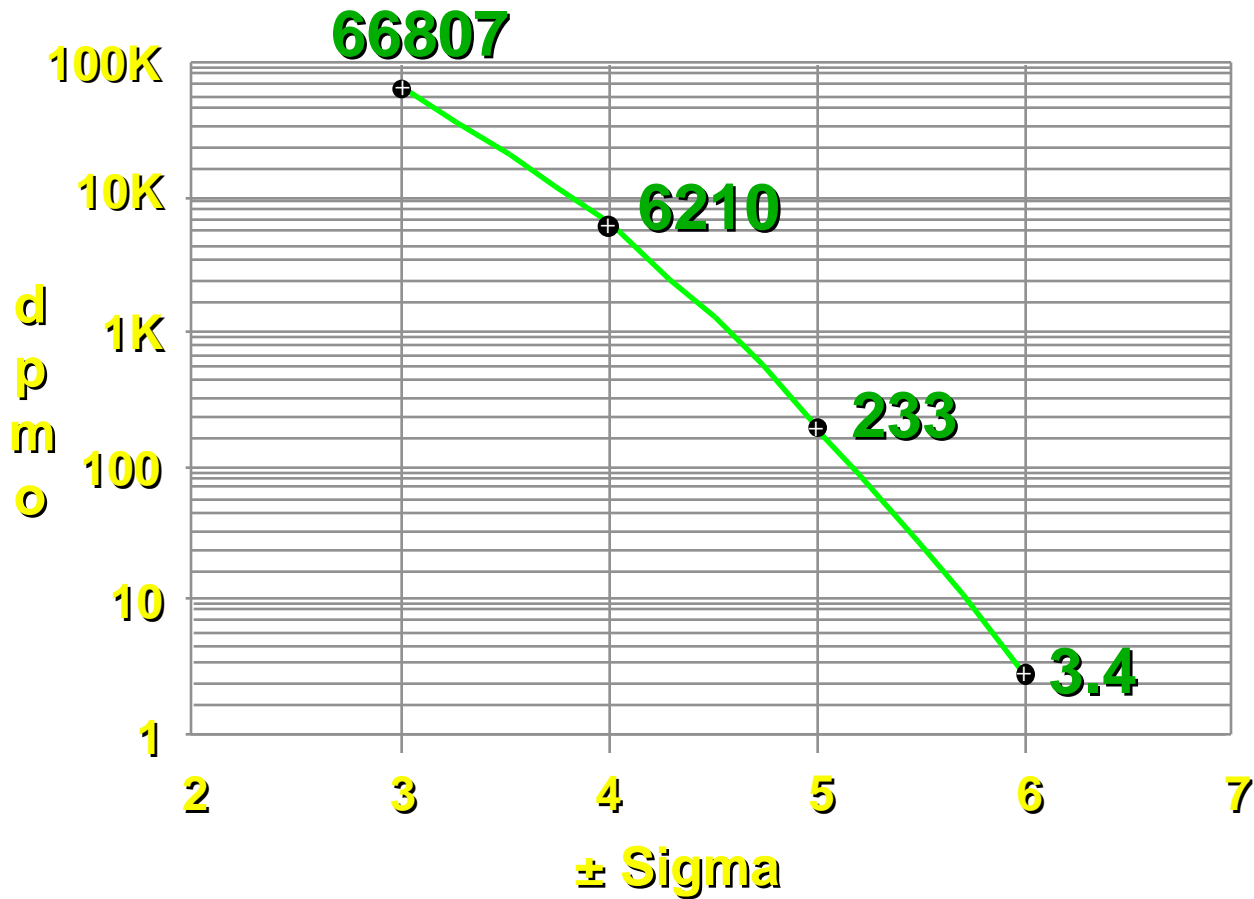
## Opportunities per Unit

- For rational comparison of quality, the complexity must be considered.
- Opportunitites are a measure of complexity.
- “Opportunities” are the number of things that must be right for the unit to meet all the requirements.
- Examples: features, characteristics, number of components, square meters, etc.

# Defects per million opportunities

$$\text{dpmo} = \frac{\text{DPU}}{\text{opportunities per unit}} \times 10^6$$

# dpmo to Sigma Relationship



# Six Sigma Product Design

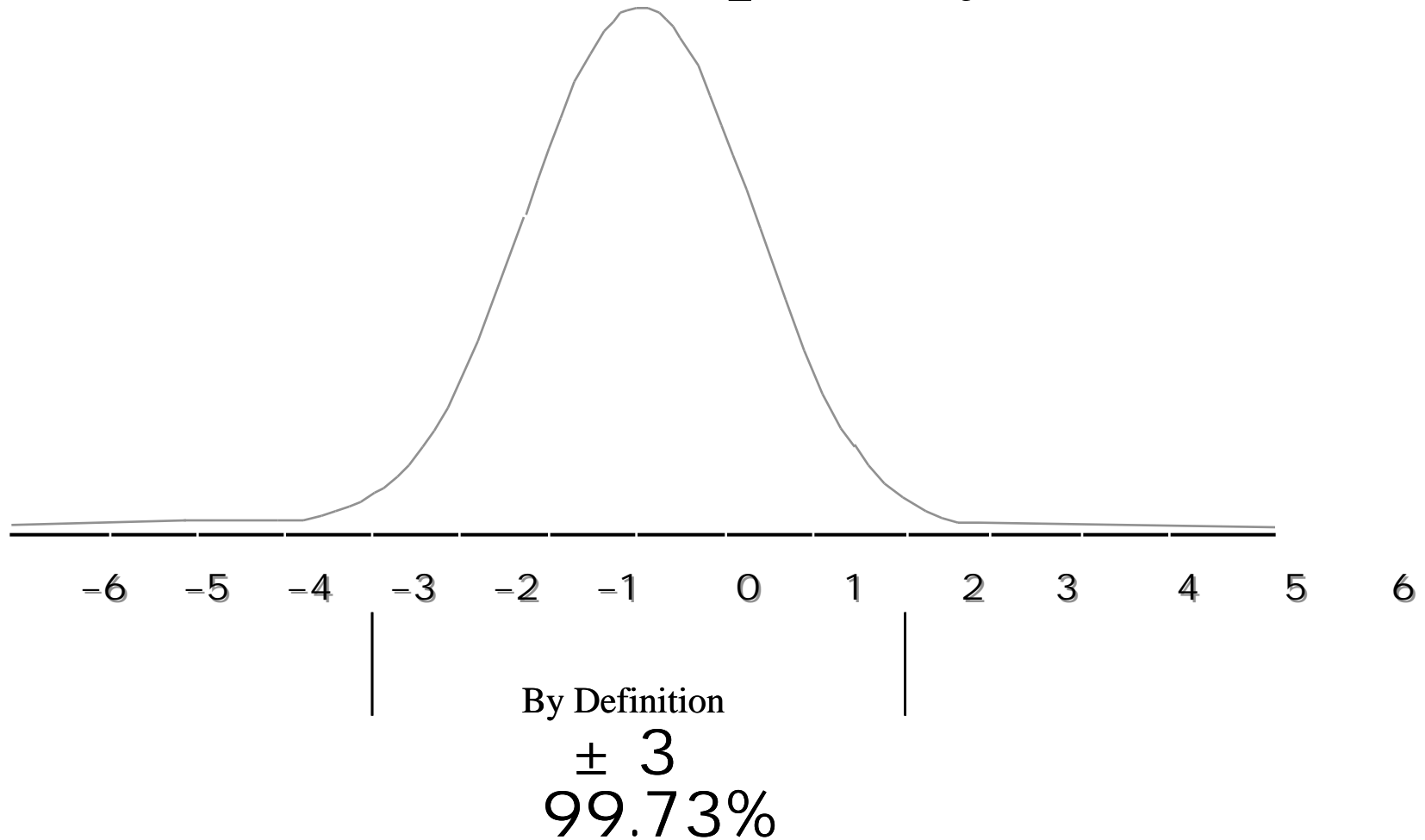
*Six Sigma design is the application of statistical techniques to analyze and optimize the inherent system design margins. The objective is a design which can be built error-free.*

Rich Karm  
Ron Randall

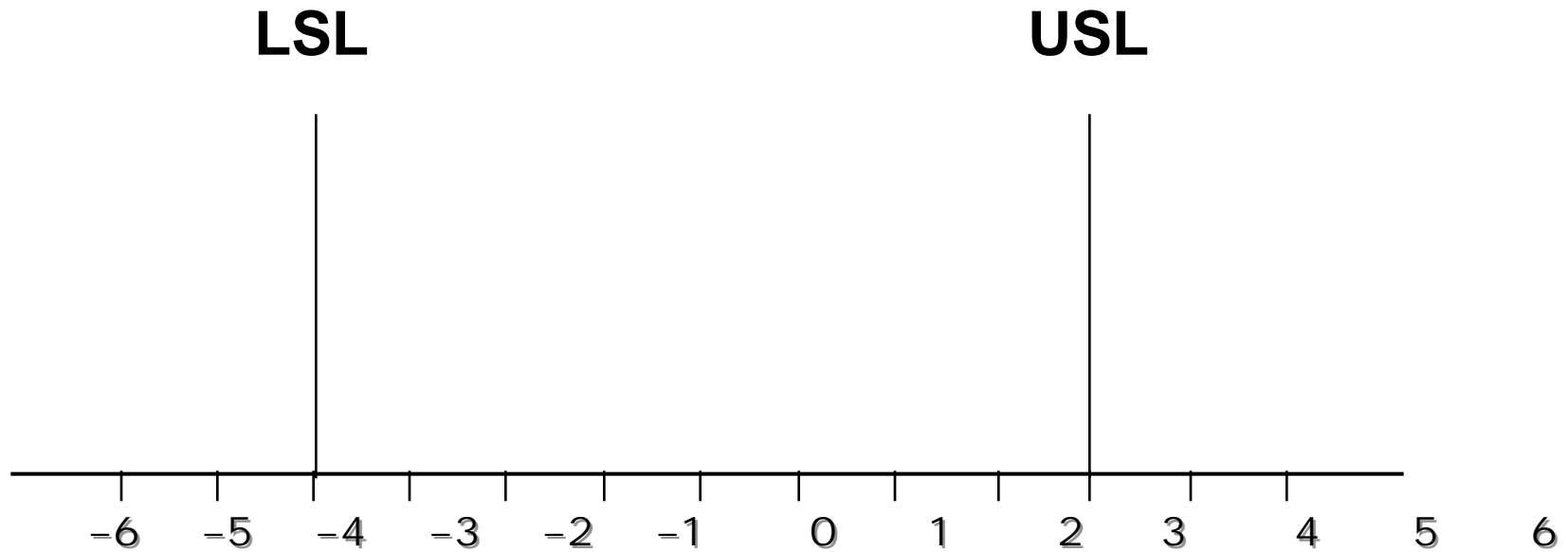
# **Six Steps For Integrated Product/Process Development Teams**

1. Identify the customer's physical and functional requirements.
2. Determine the product characteristics critical to each requirement.
3. Determine if the characteristic is controlled by the part, the process, or both.
4. Determine the target nominal and maximum tolerance allowable for each characteristic.
5. Determine the process capability for each critical characteristic.
6. If  $C_p < 2$  OR  $C_{pk} < 1.5$ , redesign the product or process as required.

# Process Capability



# Specification Limits



# Process Capability Index

## C<sub>p</sub>

By Definition:

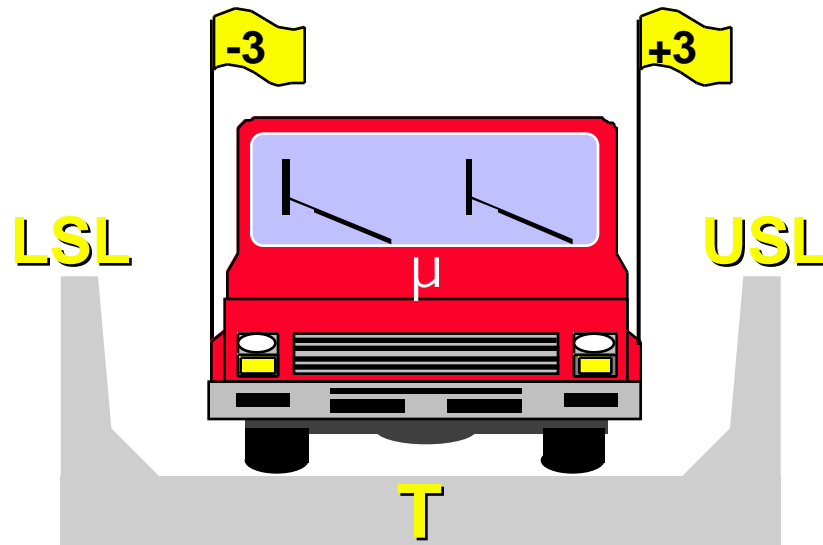
$$C_p = \frac{\text{Spec Width (road)}}{\text{Mfg Capability (car)}} = \frac{\text{USL} - \text{LSL}}{\pm 3}$$

*Concurrent Engineering Index*

*Design / Manufacturing*

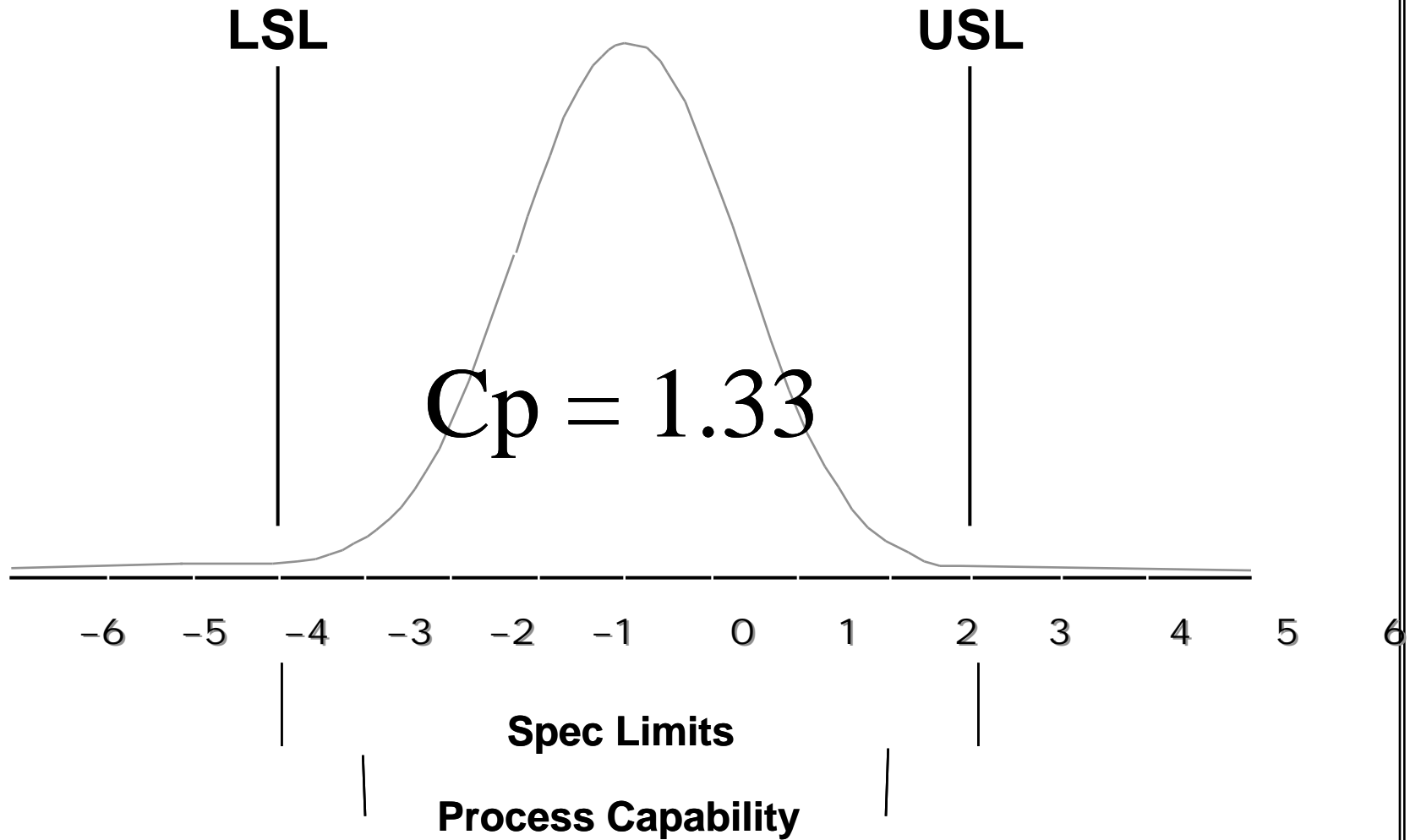


# Capability Index



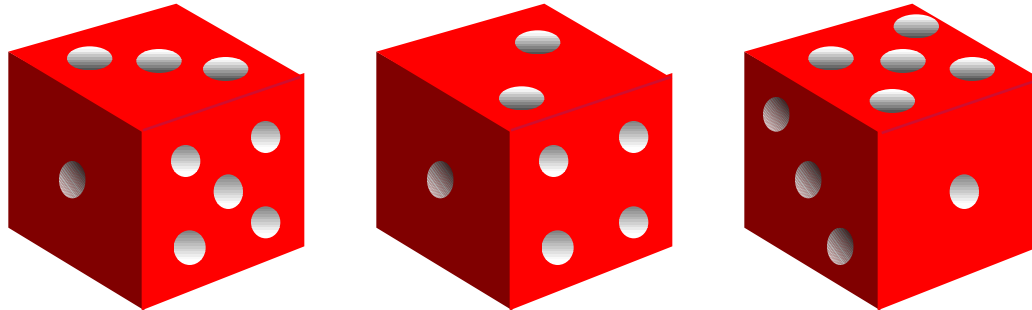
$$C_p = \frac{\text{Spec Width (road)}}{\text{Mfg Capability (car)}} = \frac{USL - LSL}{\pm 3}$$

# Capability Index at $\pm 4$ Sigma



# Combined Probability of Independent Events

Probability of not throwing a 1 on any of 3 die:




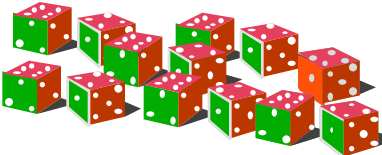


$$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} =$$

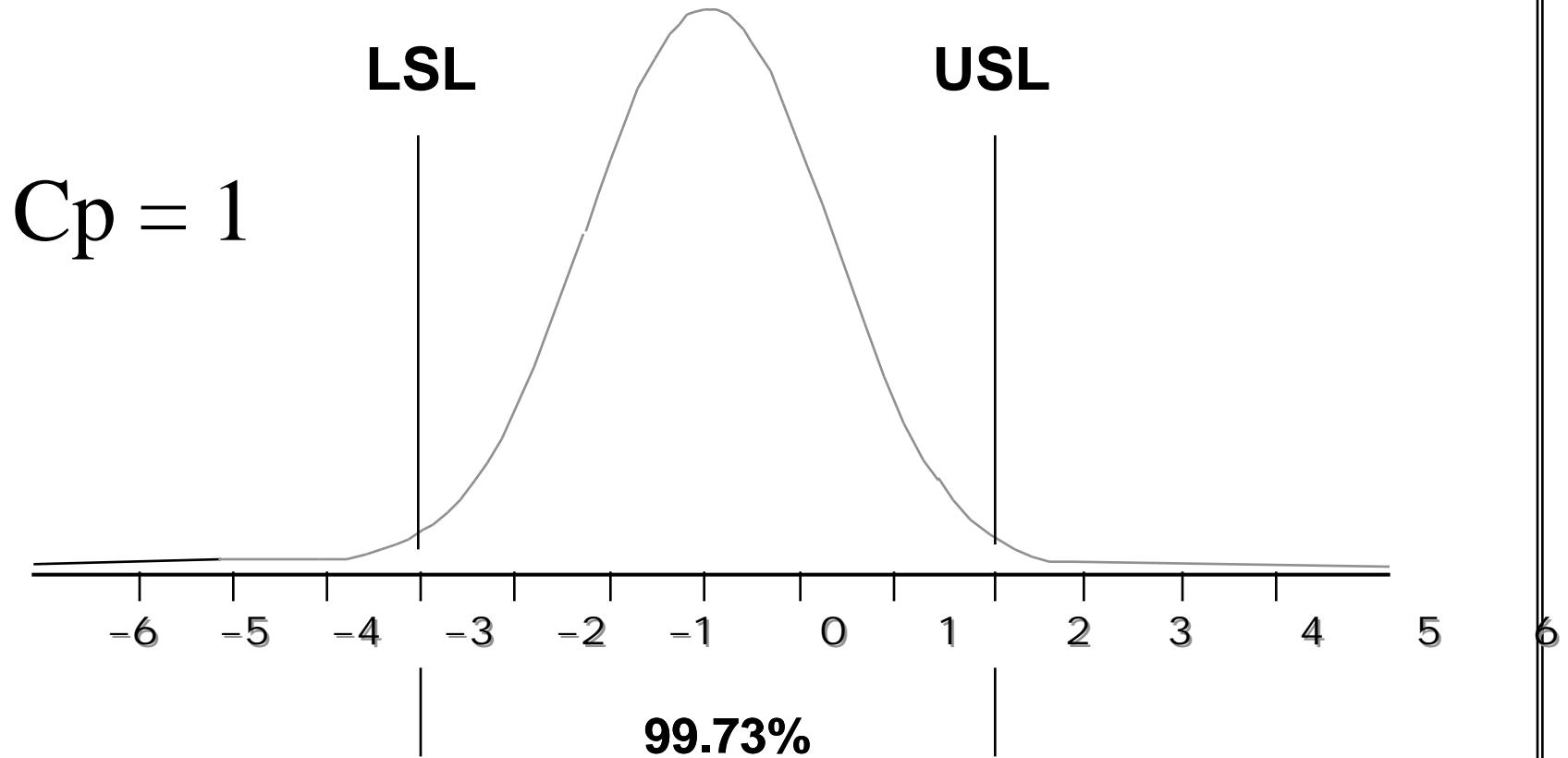
$$57.87\% =$$

Rolled Yield

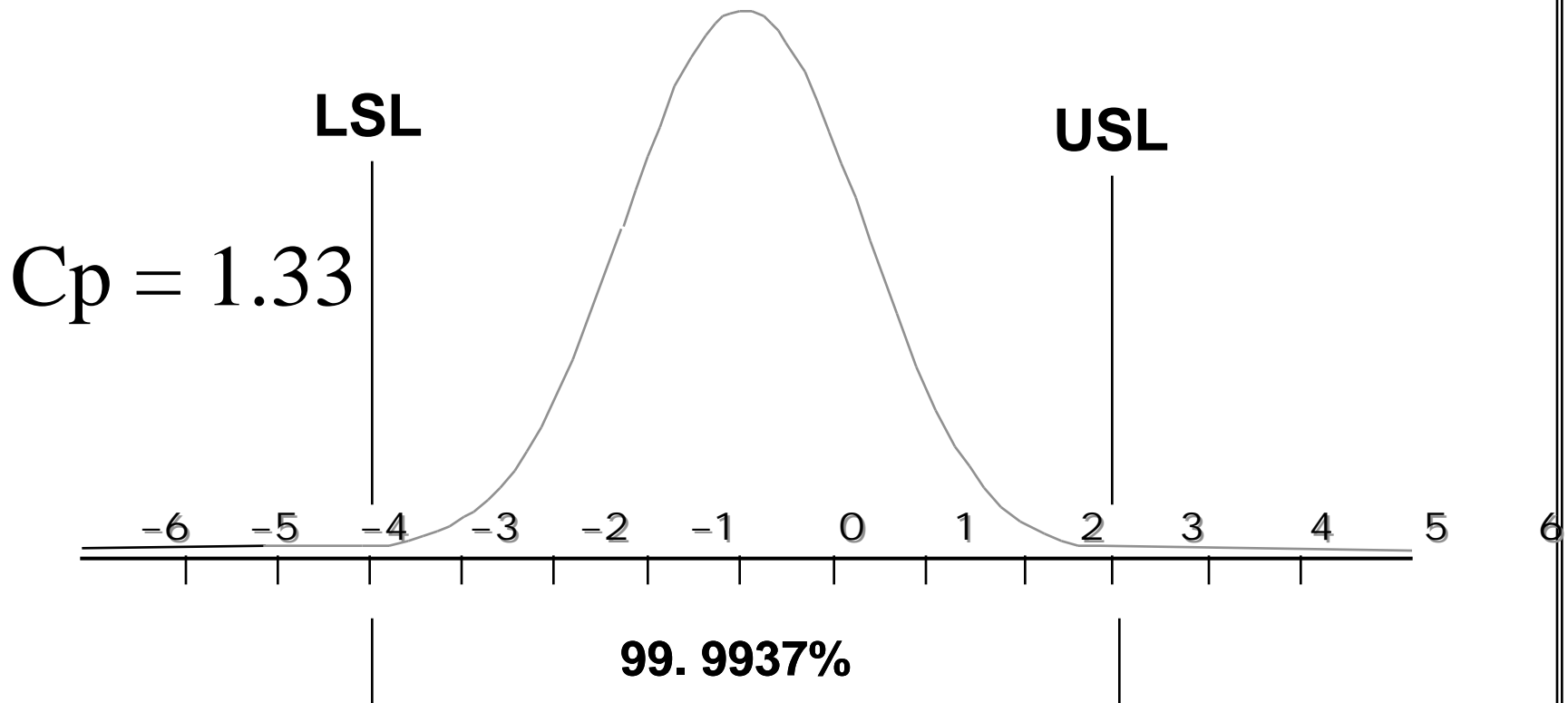
# Probability of Not Rolling a One

	# of dice (x)	$RY = Y_1^x$
	1	83.33%
	3	57.87%
	10	16.15%
	30	.42%
	50	.01%
	100	--

# Probability of Success with $\pm 3$ Sigma Spec Limits



# Probability of Success with $\pm 4$ Sigma Spec Limits



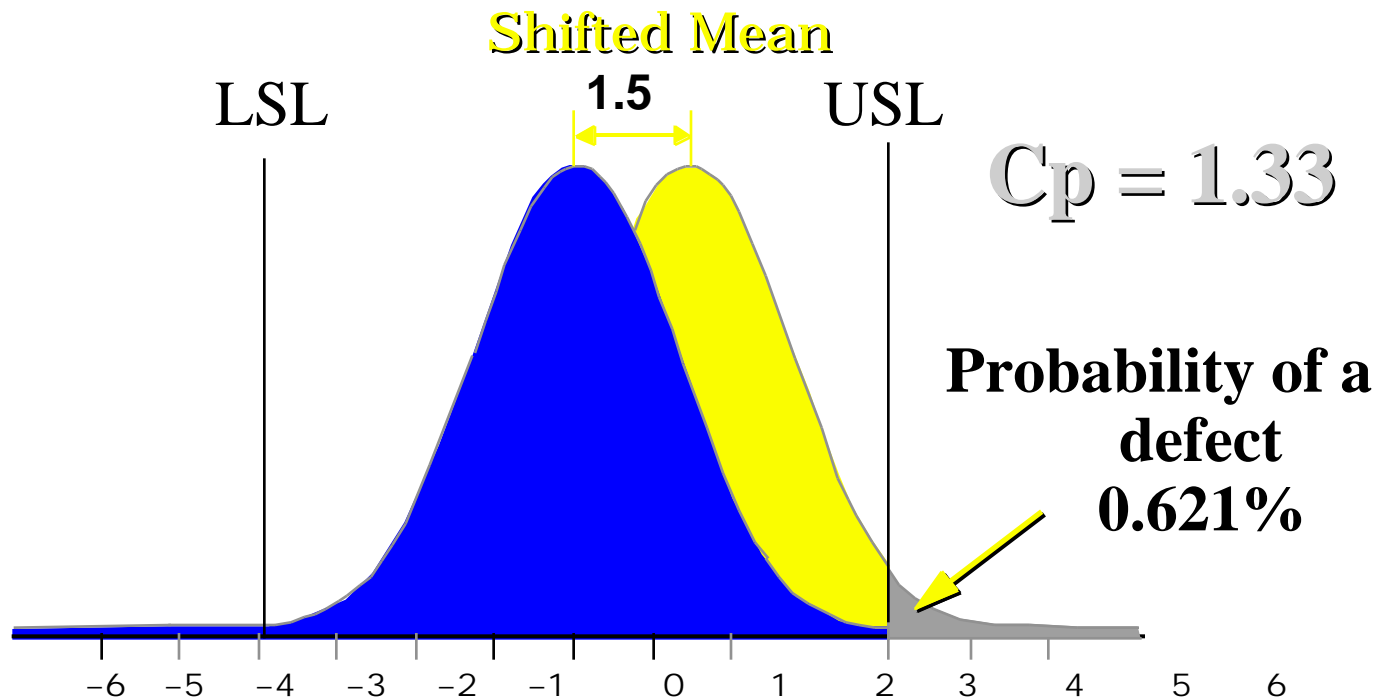
# Spec Limit Effect on Yields

(If distribution is always centered)

# of parts or steps (x)	Spec Limits at		$RY = Y_1^x$
	$\pm 3$	$\pm 4$	
1	99.73%	99.9937%	
10	97.33	99.94	
30	92.21	99.81	
50	87.36	99.68	
100	76.31	99.37	
150	66.66	99.06	
200	58.23	98.74	
300	44.44	98.12	
400	33.91	97.50	
500	25.88	96.89	

# Reality

## Processes are not always in the center





**Cpk**

**Process capability adjusted for centering**

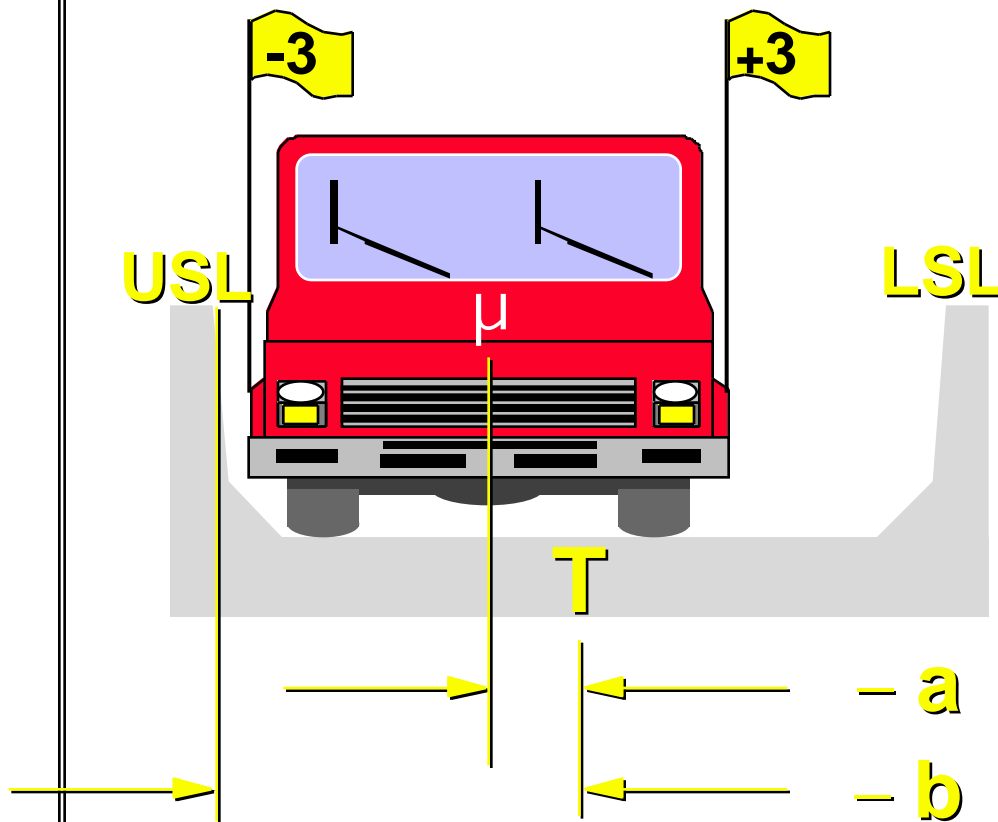
**By Definition:**

$$\mathbf{Cpk = Cp(1-k)}$$

$$\mathbf{k = amount\ of\ shift / 1/2(USL-LSL)}$$

**k is always positive**

# Capability Index Adjusted for Shifted Mean



$$C_{pk} = C_p (1-k)$$

where  $k = \frac{-a}{-b}$

$k$  is always  
positive

# $\pm 4$ Sigma Spec Width

Shifted Mean

$$C_p = 1.33$$

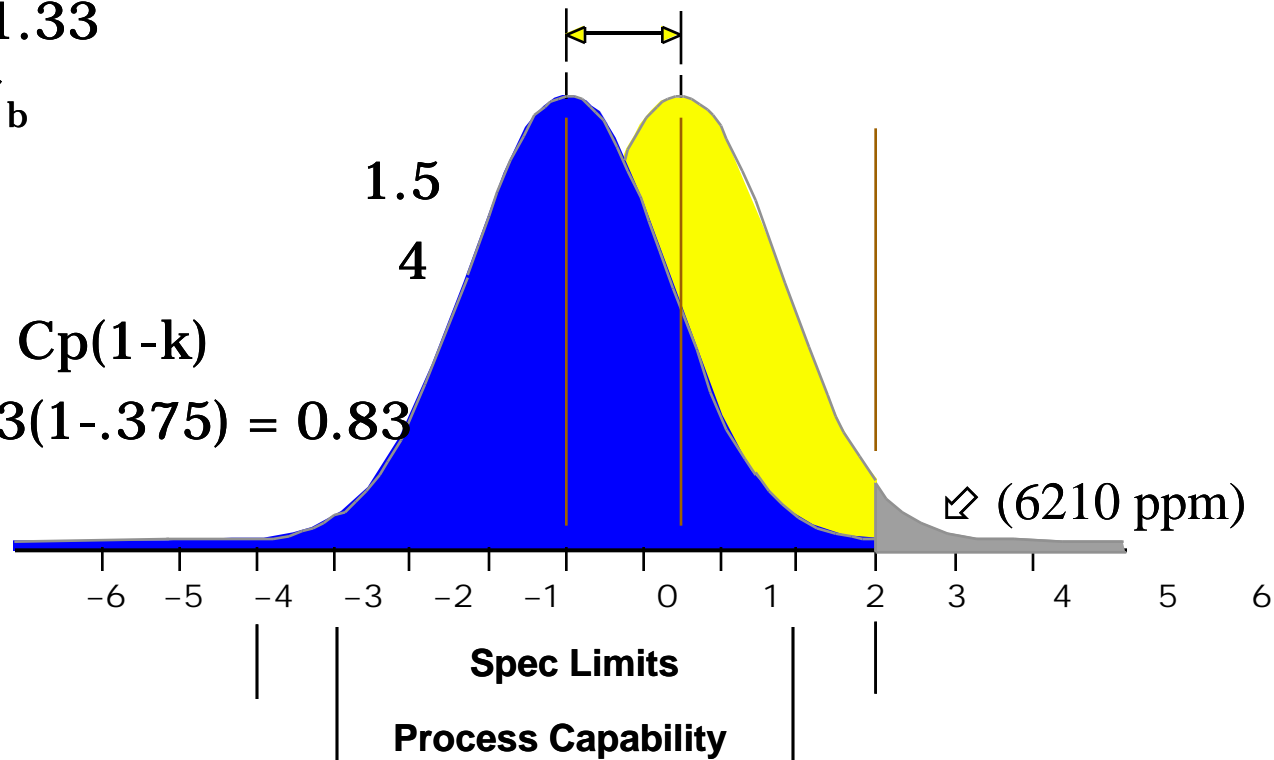
$$k = a/b$$

$$a =$$

$$b =$$

$$C_{pk} = C_p(1-k)$$

$$= 1.33(1-.375) = 0.83$$



# $\pm 6$ Sigma Spec Width

Shifted Mean

$$C_p = 2$$

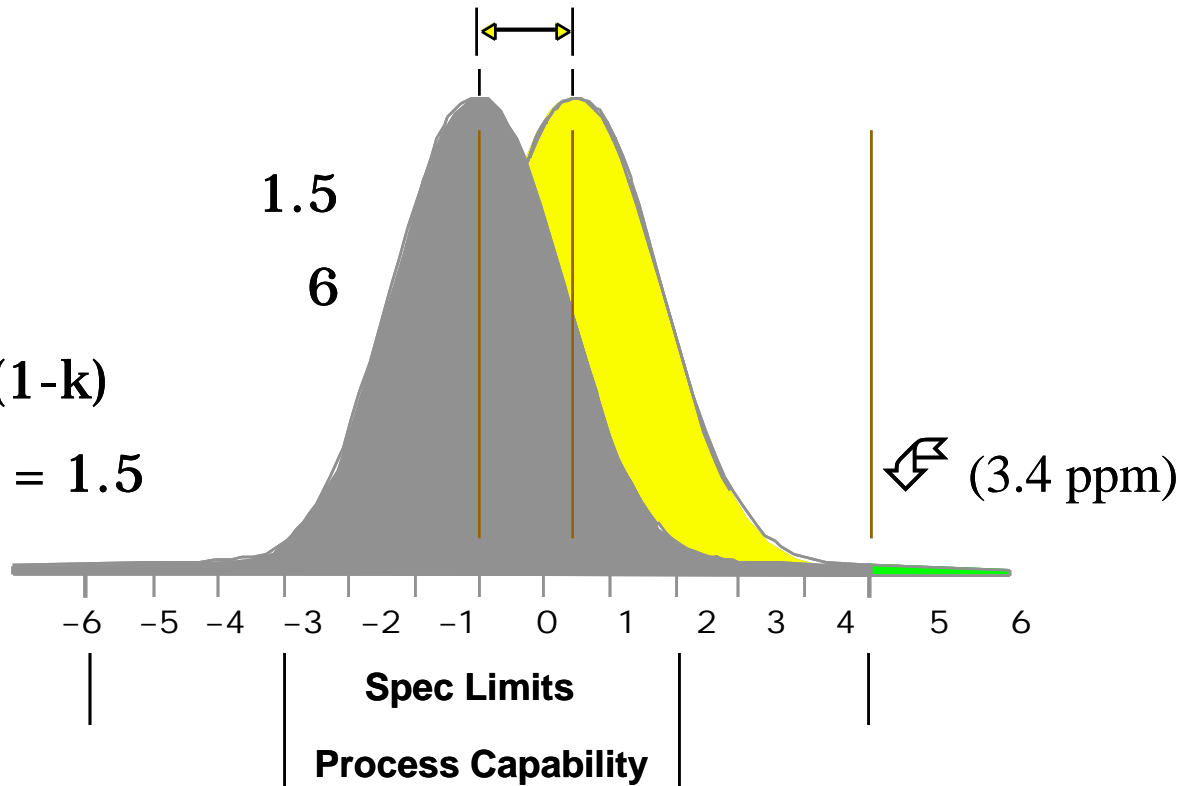
$$k = a/b$$

$$a =$$

$$b =$$

$$C_{pk} = C_p(1-k)$$

$$= 2(1-.25) = 1.5$$

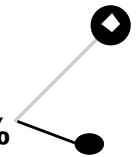


# Yields thru Multiple Steps/Parts

Distribution Shifted +/- 1.5

$$RY = Y_1^x$$

x # of parts or steps	± 3	± 4	± 5	± 6
1	93.32%	99.379%	99.9767%	99.99966%
10	50.08	93.96	99.768	99.9966
30	12.57	82.95	99.30	99.99
50	---	73.24	98.84	99.98
100	---	53.64	97.70	99.966
150	---	39.38	96.61	99.949
200	---	28.77	95.45	99.932
300	---	15.43	93.26	99.898
400	---	8.28	91.11	99.864
500	---	4.44	89.02	99.830
800	---	00.69	83.02	99.729
1200	---	00.06	75.88	99.593



# The Technical Feasibility of Unobservium

## An Exercise in Cp and Cpk

## Cp and Cpk Exercise

- Your mission is to develop a practical invisible aircraft using a new material.
- Introducing - Unobservium!
- Unobservium is an electroluminescent material with the strength to weight characteristics which make it ideal for aircraft structural use.
- Your job is to evaluate the critical characteristics of laboratory samples of unobservium statistically, and recommend where research money should be invested to reduce the risk of transition to production.

## Cp and Cpk Exercise

- Unobservium responds to the application of electrical current by absorbing or emitting electromagnetic radiation throughout the visible and much of the invisible electromagnetic spectrum, including the infrared and microwave spectrums.



## Cp and Cpk Exercise

- This allows an object covered with unobservium to change the light absorption and reflection of its surface, rendering it invisible to optical, laser, and radar tracking devices.
- This is accomplished through the use of a set of classified sensors and electronic stimulators controlled by very fast microprocessors running a sophisticated algorithm nicknamed “chameleon”.

## Cp and Cpk Exercise

- Research has found that there are three critical characteristics of the material:
  1. Reflectivity
  2. Yield strength
  3. Hardness

## Cp and Cpk Exercise

- All specifications are double sided. Each has a high and a low spec limit.
  1. Reflectivity: Too much and the aircraft becomes visible because it is shiny. Too little and it appears dark against the background.
  2. Yield strength: Too much and metal fatigue problems become pronounced. Too little and the material tears.

## Cp and Cpk Exercise

3. Hardness: Too much and the material cannot be economically machined and formed. Too little and the material abrades in flight causing the reflectivity to change.

## Cp and Cpk Exercise

- The class will divide into teams.
- Each team will gather data on one of the critical characteristics from laboratory samples.
- Each team will then compare its data against the functional spec limits for that material characteristic.

# Cp and Cpk Exercise

## Hardness

This exercise simulates gathering hardness data from 40 samples of unobservium from the laboratory.

- Throw a pencil at a flip chart.
- The target is a vertical line in the middle of the chart.
- The throwers must stand 10 ft. (3 meters) away from the target.
- The team must record at least 40 throws. Each throw represents a sample hardness reading.

# Cp and Cpk Exercise

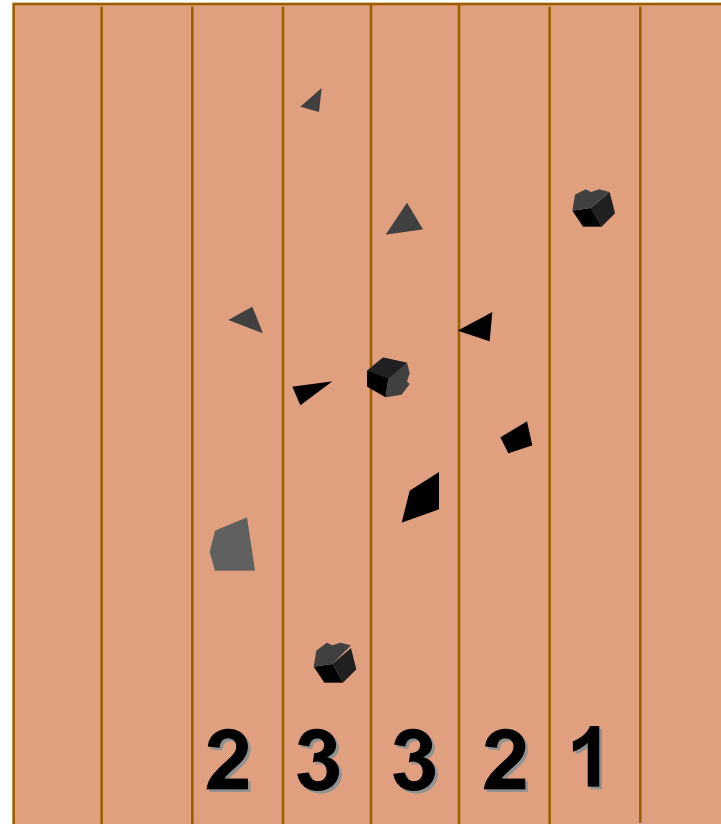
## Hardness

- Count the number of pencil hits (marks) up to one inch to the right of the target, one inch to the left of the target, between one and two inches from the target on each side, between two and three inches from the target on each side, etc.

# Cp and Cpk Exercise

## Hardness

- Write the total number of hits in each column at the bottom of each column.
- A column is the area between the lines.

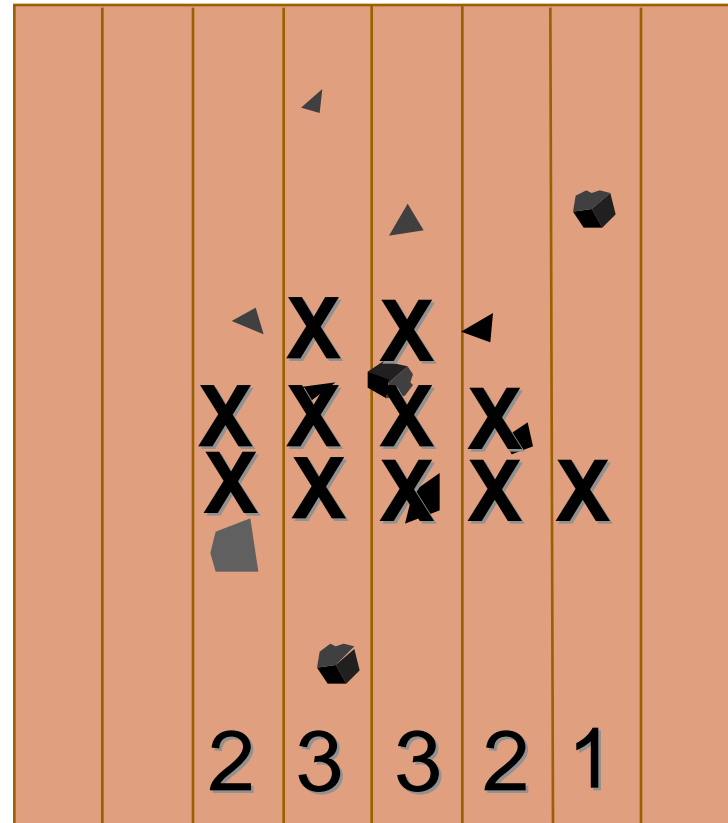




# Cp and Cpk Exercise

## Hardness

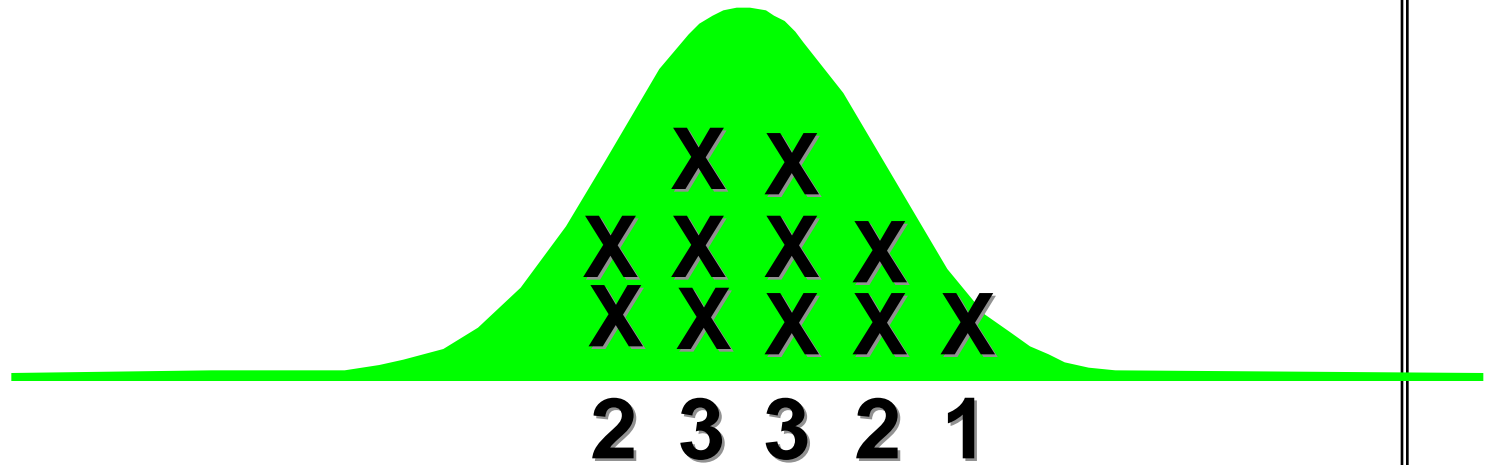
- Create a histogram using the number of hits in each column.



# Cp and Cpk Exercise

## Hardness

- Calculate the mean and standard deviation for the samples.



# Cp and Cpk Exercise

## Hardness

- Use mid-range values for each column to simplify the calculations.
- Example: 2 hits at -1.5”  
3 hits at -0.5”  
3 hits at 0.5”  
2 hits at 1.5”  
1 hit at 2.5”

**-1.5 -0.5 .5 1.5 2.5**

	<b>X</b>	<b>X</b>			
<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>		
<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	
<b>2</b>	<b>3</b>	<b>3</b>	<b>2</b>	<b>1</b>	

# Cp and Cpk Exercise

## Hardness

- For a target with a width of +/- 2 inches (+/- 5 cm.) centered on the target line, what is the Cp and Cpk?

Spec Width

$$C_p = \frac{\text{-----}}{6x(\text{Std. Dev.})}$$

# Cp and Cpk Exercise Hardness

$$Cpk = Cp (1-k)$$

$$| \text{Mean} - \text{Target} |$$

$$k = \frac{\text{-----}}{1/2 \text{ Spec Width}}$$

# Cp and Cpk Exercise

## - Some Useful Formulas -

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \bar{x}^2}$$

# Cp and Cpk Exercise

## Discussion

- Which characteristic has the lowest Cp and Cpk?
- Which characteristic is the most likely to have problems?
- Where should we devote our attention and research dollars?

# Cp and Cpk Exercise Summary

**Cp and Cpk can point out areas of design risk.**

**By focusing our technical resources on the potential problem areas we can avoid future problems and costly surprises.**

**By using six sigma techniques to identify critical variables and control them, we can reduce the risk associated with transitioning advanced technology into production.**



National Center for Advanced Technologies



Malcolm Baldrige  
**National  
Quality  
Award**

**1992  
Winner**



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***Defense Systems &  
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